

Mathematical Reasoning

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Disciplinary Reasoning

Students working within this GP will develop the following disciplinary reasoning skills over time.

- Making sense of problems and persevere in solving them.
- Constructing viable arguments and critique the reasoning of others.
- Modeling with mathematics.
- Using appropriate tools strategically.
- Looking for and making use of structure.
- Looking for and expressing regularity in repeated reasoning.

Algebraic Reasoning

* Widely Applicable Prerequisite
+Modeling Standard

MR.A.1
Interpret the structure of expressions.*+

MR.A.2
Create equations that describe numbers or relationships.*+

- Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

Functional Reasoning

* Widely Applicable Prerequisite
+Modeling Standard

MR.F.1
Understand the concept of a function and use function notation.*

- Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MR.F.2
Interpret functions that arise in applications in terms of the context.**

- For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs

MR.A.3 Understand solving equations as a process of reasoning and explain the reasoning.* - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. MR.A.4 Solve equations and inequalities in one variable.* - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. MR.A.5 Solve systems of equations.* - Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. MR.A.6 Represent and solve equations and inequalities graphically.* - Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. MR.A.7 Write expressions in equivalent forms to solve problems. - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. - Factor a quadratic expression to reveal the zeros of the function it defines. - Use the properties of exponents to transform expressions for exponential functions.	showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. MR.F.3 Analyze functions using different representations.*+ - Graph linear and quadratic functions and show intercepts, maxima, and minima. - Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. - Graph exponential and logarithmic functions, showing intercepts and end behavior. MR.F.4 Build a function that models a relationship between two quantities.* - Write a function that describes a relationship between two quantities. MR.F.5 Build new functions from existing functions. - Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. MR.F.6 Construct and compare linear, quadratic, and exponential models and solve problems.*+ - Distinguish between situations that can be modeled with linear functions and with exponential functions. - For exponential models, express as a logarithm the solution to $ab^{at} = d$ where
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MR.A.8 Perform arithmetic operations on polynomials. - Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. MR.A.9 Understand the relationship between zeros and factors of polynomials. - Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. MR.F.7 Use functions (polynomial, power, rational, exponential, logarithmic and logistic) to model and solve problems; justify results. MR.F.8 Use the composition and inverse of functions to model and solve problems. MR.F.9 Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts. MR.F.10 Develop and use the law of sines and the law of cosines and inverse trigonometric functions. MR.F.11 Operate with vectors in two dimensions to model and solve problems. MR.F.12 Expand the concept of functions to include the analysis and interpretation of both continuous and discontinuous functions in problem situations and the development of the concept of limit. MR.F.13 Extend the concept of slope of a line to develop the concept of derivative. MR.F.14 Apply techniques of antidifferentiation. MR.F.15 Convert between general and standard forms of equations of conic sections, identify whether an equation of a conic section describes a parabola, a circle, a non circular ellipse, or a hyperbola by examining its discriminant.
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Career and College Ready		
Quantitative Reasoning * Widely Applicable Prerequisite +Modeling Standard	Geometric Reasoning * Widely Applicable Prerequisite +Modeling Standard	Statistical Reasoning * Widely Applicable Prerequisite +Modeling Standard
MR.Q.1 Reason quantitatively and use units to solve problems. MR.Q.2 Use complex numbers in polynomial identities and equations. - Solve quadratic equations with real coefficients that have complex solutions. - Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	MR.G.1 Experiment with transformations in the plane* - Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. - Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. MR.G.2 Prove geometric theorems* - Prove theorems about triangles. - Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. <i>(Falls under SRT cluster)</i> MR.G.3 Explain volume formulas and use them to problem solve - Use volume formulas for prisms, cylinders,	MR.S.1 Summarize, represent, and interpret data on a single count or measurement variable* - Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. MR.S.2 Summarize, represent, and interpret data on two categorical and quantitative variables* - Fit a linear function for a scatter plot that suggests a linear association. MR.S.3 Interpret linear models** - Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. MR.S.4 Make inferences and justify conclusions from sample surveys, experiments, and observational studies* WSD.HS.MR.S.5 - Statistical.5 Understand independence and conditional probability and use them to interpret data

	pyramids, cones, and spheres to solve problems MR.G.4 Define trigonometric ratios and solve problems involving right triangles • Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.* MR.G.5 Understand and apply theorems about circles • Identify and describe relationships among inscribed angles, radii, and chords MR.G.6 Understand similarity in terms of similarity transformations • Verify experimentally the properties of dilations given by a center and a scale factor • Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. MR.G.7 Use coordinates to prove simple geometric theorems algebraically • Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	WSD.HS.MR.S.6 - Statistical.6 Use the rules of probability to compute the probability of compound events WSD.HS.MR.S.7 - Statistical.7 Use probability to evaluate outcomes of decisions or problem solve
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	MR.G.8 Apply geometric concepts in modeling situations • Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	
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Approaching					
	Geometry	Functions	Expressions & Equations	The Number System	
8	MR.G.8.1 Understand congruence and similarity using physical models, transparencies, or geometry software. - Verify experimentally the properties of rotations, reflections, and translations: - Lines are taken to lines, and line segments to line segments of the same length. - Angles are taken to angles of the same measure. - Parallel lines are taken to parallel lines. - Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. - Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. - Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. - Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. MR.G.8.2 Understand and apply the Pythagorean Theorem. - Explain a proof of the Pythagorean Theorem	MR.F.8.1 Define, evaluate, and compare functions. - Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. - Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). - Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. MR.F.8.2 Use functions to model relationships between quantities. - Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. - Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	MR.EE.8.1 Work with radicals and integer exponents - Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $32 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. - Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. - Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology MR.EE.8.2 Understand the connections between proportional relationships, lines, and linear equations - Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	MR.NS.8.1 Know that there are numbers that are not rational, and approximate them by rational numbers	

	and its converse. • Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.		• Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. MR.EE.8.3 Analyze and solve linear equations and pairs of simultaneous linear equations. MR.EE.8.4 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. MR.EE.8.5 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. MR.EE.8.6 Solve linear equations in one variable. MR.EE.8.7 Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). MR.EE.8.8 Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. ○ MR.EE.8.8.1- Analyze and solve pairs of simultaneous linear equations.	
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			MR.EE.8.9 Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. MR.EE.8.10 Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. MR.EE.8.11 Solve real-world and mathematical problems leading to two linear equations in two variables.	
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	Ratios and Proportional Reasoning	The Number System	Expressions and Equations	Geometry
7	MR.RP.7.1 Analyze proportional relationships and use them to solve real-world and mathematical problems. • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. • Recognize and represent proportional relationships between quantities. ◦ Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. ◦ Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. ◦ Represent proportional relationships by equations. ◦ Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate. • CCSS.Math.Content.7.RP.A.2 Recognize and represent proportional relationships between quantities. • MR.RP.7.6 Use proportional relationships to solve multistep ratio and percent problems. MR.RP.7.2 CCSS.Math.Content.7.RP.A.2.a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing	Apply and extend previous understandings of operations with fractions. MR.NS.7.2 • Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. ◦ Describe situations in which opposite quantities combine to make 0. ◦ Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. ◦ Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. ◦ Apply properties of operations as strategies to add and subtract rational numbers. MR.NS.7.3 • Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. ◦ Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive	MR.EE.7.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. • Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. MR.EE.7.2 Solve real-life and mathematical problems using numerical and algebraic expressions and equations. • Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. • Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. ◦ Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in	MR.G.7.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

	whether the graph is a straight line through the origin. MR.RP.7.3 CCSS.Math.Content.7.RP.A.2.b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. MR.RP.7.4 CCSS.Math.Content.7.RP.A.2.c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$. MR.RP.7.5 CCSS.Math.Content.7.RP.A.2.d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.	property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. - Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. - Apply properties of operations as strategies to multiply and divide rational numbers. - Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. - Solve real-world and mathematical problems involving the four operations with rational numbers.¹	- each approach. - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. MR.EE.7.3 Use properties of operations to generate equivalent expressions. - Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. MR.EE.7.4 Use variables to represent quantities in real-world or mathematical problems, and construct simple equations and inequalities to solve problems by reasoning about the quantities. MR.EE.7.5 Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	
	Statistics & Probability			
	MR.SP.7.1 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. MR.SP.7.2 Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the			

	outcomes in the sample space which compose the event.			
	Ratios & Proportions	Number Systems	Expressions & Equations	Geometry
6	MR.RP.6.1 Understand ratio concepts and use ratio reasoning to solve problems. - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. - Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. - Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. - Solve unit rate problems including those involving unit pricing and constant speed. - Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent. - Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	MR.NS.6.1 Apply and extend previous understandings of multiplication and division to divide fractions by fractions. - Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. MR.NS.6.2 Apply and extend previous understandings of numbers to the system of rational numbers. - Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. - Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. - Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite. - Understand signs of numbers in ordered pairs as indicating locations in	MR.EE.6.1 Apply and extend previous understandings of arithmetic to algebraic expressions. - Write and evaluate numerical expressions involving whole-number exponents. - Write, read, and evaluate expressions in which letters stand for numbers. - Write expressions that record operations with numbers and with letters standing for numbers. - Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. - Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). - Apply the properties of operations to generate equivalent expressions. - Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). MR.EE.6.2 Reason about and solve one-variable equations and inequalities.	MR.G.6.1 Solve real-world and mathematical problems involving area, surface area, and volume. - Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. - Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. - Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. - Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

		quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. - Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. MR.NS.6.3 Understand ordering and absolute value of rational numbers. 6.NS.C.7.A Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. 6.NS.C.7.B Write, interpret, and explain statements of order for rational numbers in real-world contexts. 6.NS.C.7.C Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. 6.NS.C.7.D Distinguish comparisons of absolute value from statements about order. 6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. MR.NS.6.4 Compute fluently with multi-digit numbers and find common factors and multiples. - Fluently divide multi-digit numbers using the standard algorithm.	- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. - Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. - Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. - Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. MR.EE.6.3 Represent and analyze quantitative relationships between dependent and independent variables. - Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times,	
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		- Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. - Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express $36 + 8$ as $4(9 + 2)$.</i>	and write the equation $d = 65t$ to represent the relationship between distance and time.	
	DEVELOPING - Elementary Standards embedded for use in the middle school (special education use only)			
	WSD.MS.MR.MD3.2 Measurement and Data 3.2 Understand concepts of area and relate area to multiplication and to addition	WSD.MS.MR.NOF.4.3 Number and Operations Fractions 4.3 Understand decimal notation for fractions and compare decimal fractions	WSD.MS.MR.NOBT.5.1 Number and Operations in Base Ten 5.1 Understand the place value system	WSD.MS.MR.OA.3.4 Operations and Algebraic Thinking 3.4 Solve problems involving the four operations, and identify and explain patterns in arithmetic.
	WSD.MS.MR.NOF.3.1 Number and Operations Fractions 3.1 Develop understanding of fractions as numbers	WSD.MS.MR.NOF.5.1 Number and Operations Fractions 5.1 Use equivalent fractions as a strategy to add and subtract fractions	WSD.MS.MR.OA.3.1 Operations and Algebraic Thinking 3.1 Represent and solve problems involving multiplication and division.	WSD.MS.MR.OA.4.1 Operations and Algebraic Thinking 4.1 Use the four operations with whole numbers to solve problems.
	WSD.MS.MR.NOF.4.1 Number and Operations Fractions 4.1 Extend understanding of fractional equivalence and ordering	WSD.MS.MR.NOF.5.2 Number and Operations Fractions 5.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions	WSD.MS.MR.OA.3.2 Operations and Algebraic Thinking 3.2 Understand properties of multiplication and the relationship between multiplication and division.	
	WSD.MS.MR.NOF.4.2 Number and Operations Fractions 4.2 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers		WSD.MS.MR.OA.3.3 Operations and Algebraic Thinking 3.3 Fluently multiply and divide within 100.	

Developing			
	Number and Operations in Base Ten	Numbers and Operations - Fractions	Measurement and Data
5	MR.NOBT.5.1 Understand the place value system - Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. - Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. - Use place value understanding to round decimals to any place MR.NOBT.5.2 Perform operations with multi-digit whole numbers and with decimals to hundredths. -	MR.NOF.5.1 Use equivalent fractions as a strategy to add and subtract fractions - Add and subtract fractions with unlike denominators - Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators MR.NOF.5.2 Apply and extend previous understandings of multiplication and division to multiply and divide fractions - Interpret a fraction as division of the numerator by the denominator - Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction - Interpret multiplication as scaling (resizing) - Solve real world problems involving multiplication of fractions and mixed numbers - Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions	MR.MD.5.1 Understand concepts of volume and relate volume to multiplication and to addition. - Recognize volume as an attribute of solid figures and understand concepts of volume measurement - Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units - Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume
	Operations and Algebraic Thinking	Numbers and Operations in Base Ten	Numbers and Operations - Fractions
4	MR.OAT.4.1 Use the four operations with whole numbers to solve problems - Interpret a multiplication equation as a comparison - Multiply or divide to solve word problems involving multiplicative comparison - Solve multi-step word problems posed with whole numbers and having whole number answers, using the four operations	MR.NOBT.4.1 Generalize place value understanding for multi digit whole numbers - Recognize that in a multi-digit whole number, a digit in one place represents tens times its place to the right - Read and write multi-digit whole numbers using base ten numerals, number names and expanded form - Use place value understanding to round multi digit whole numbers to any place	MR.NOF.4.1 Extend understanding of fractional equivalence and ordering - Explain why a fraction is equivalent to another fraction - Compare two fractions with different numerators and different denominators MR.NOF.4.2 Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

		MR.NOBT.3-4.2 Use place value understanding and properties of operations to perform multi-digit arithmetic • Fluently add and subtract multi-digit whole numbers using the standard algorithm • Multiply a whole number of up to four digits by a one digit whole number and multiply a 2 digit by 2 digit number	• Understand a fraction a/b with $a > 1$ as a total of fractions $1/b$ • Apply and extend previous understandings of multiplication to multiply a fraction by a whole number MR.NOF.4.3 Understand decimal notation for fractions and compare decimal fractions • Express a fraction with denominator 10 as an equivalent fraction denominator 100 and use this technique to add two fractions • Use decimal notation for fractions with denominators 10 or 100 • Compare two decimals to 100ths by reasoning about their size
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	Operations and Algebraic Thinking	Numbers and Operations - Fractions	Measurement and Data
3	MR.OAT.3.1 Represent and solve problems involving multiplication and division - Interpret products of whole numbers - Interpret whole number quotients of whole numbers - Use multiplication and division within 100 to solve word problems - Determine the unknown whole number in a multiplication or division equation relating three whole numbers MR.OAT.3.2 Understand properties of multiplication and the relationship between multiplication and division - Apply properties of operations as strategies to multiply and divide - Understand division as an unknown factor problem MR.OAT.3.3 Fluently multiply and divide within 100 MR.OAT.3.4 Solve problems involving the 4 operations, and identify and explain patterns in arithmetic - Solve two step word problems involving the four operations - Identify arithmetic patterns and explain them using the four operations	MR.NOF.3.1 Develop understanding of fractions as numbers - Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$. - Understand and represent fractions on a number line - Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size	MR.MD.3.1 Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. - Tell and write time to the nearest minute and measure time intervals in minutes - Measure and estimate liquid volumes and masses of objects using standard units MR.MD.3.2 Understand concepts of area and relate area to multiplication and to addition. - Recognize area as an attribute of plane figures and understand concepts of area measurement - Measure areas by counting unit squares - Relate area to the operations of multiplication and addition MR.MD.3.3 Represent and interpret data. - Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. - Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. - Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
		Numbers and Operations in Base Ten	
		(note that there is no ".1" here in order to match the numbering in grade 4)	

		MR.NOBT.3-4.2 Use place value understanding and properties of operations to perform multi-digit arithmetic. • Use place value understanding to round whole numbers to the nearest 10 or 100. • Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. • Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.	
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Foundational			
	Operations and Algebraic Thinking	Number and Operations in Base Ten	Measurement and Data
2	MR.OAT.1-2.1 Represent and solve problems involving addition and subtraction - use addition and subtraction within 100 to solve 1 and 2 step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all spots, using drawings and equations with a symbol for the unknown to represent the problem MR.OAT.1-2.2 Add and subtract within 20 - fluently add and subtract within 20 using mental strategies. By the end of grade 2, know from memory the totals of all 2 one digit numbers	(note that there is no ".1" here in order to match the numbering in grade 1) MR.NOBT.1-2.2 Understand place value - understands that 3 digit numbers are hundreds, tens and ones 100 is a bundle of ten 10s - The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds - count within 1000 (skip count by 5s, 10s, 100s) - read and write numbers to 1000 using base ten, number names and expanded form - compare two 3 digit numbers using $<$, $>$ or $=$ MR.NOBT.1-2.3 Use place value understanding and properties of operations to add and subtract - fluently add and subtract within 100 using place value strategies, properties of operations and relationship between addition and subtraction - add up to 4 two digit numbers using strategies based on place value and operations - add and subtract within 1000 using models or drawings based on place value, operations and relationship between addition and subtraction. Understand that with 3 digit numbers there are 100s, 10s and 1s, and sometimes it's necessary to	MR.MD.2.1 Measure and estimate lengths in standard units - measure the length of an object by selecting and using the appropriate tools - measure the length of an object twice using the length unit of different lengths - estimate lengths using inches, feet, centimeters and meters - measure to determine how much longer one object is than another MR.MD.2.2 Relate addition and subtraction to length - use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units - represent whole numbers as lengths on a number line with equally space points corresponding to 0, 1, 2 and represent whole number totals and differences within 100 on a number line MR.MD.2.3 Work with time and money - Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. - Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
1	MR.OAT.1-2.1 Represent and solve problems involving addition and subtraction use addition and subtraction to solve word problems involving adding to, taking from, putting together, taking	MR.NOBT.1.1 Extending the counting sequence count to 120 starting at any number less than 120. Read and write numerals and represent a number of objects with a	MR.MD.1.1 Measure lengths indirectly and by iterating length units order three objects by length; compare the lengths between two using the third

	apart and comparing with unknowns in all positions by using objects, drawings and equations with a symbol for the unknown. • solve word problems that add three whole numbers less than 20 by using objects, drawings and equations with a symbol for the unknown MR.OAT.1-2.2 Add and subtract within 20 • relate counting to addition and subtraction • add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as “make 10”, decomposing a number to 10, using the relationship between addition and subtraction, creating equivalent and easier known totals MR.OAT.1.3 Understand and apply properties of operations and the relationship between addition and subtraction • apply properties of operations as strategies to add and subtract • understand subtraction as an unknown addend problem MR.OAT.1.4 Work with addition and subtraction equations • understand the meaning of the equal sign, determine if addition and subtraction equations are true or false (balanced equations) • determine the unknown whole number in an addition or subtraction equation (fact families)	written numeral MR.NOBT.1-2.2 Understand place value • understand that two digits represent 10s and 1s • 10 can be thought of as a bundle of ten 1s • the numbers between 11 and 19 are composed of a 10 and some 1s • the numbers 10-90 (multiples of 10) refer to 1, 2, 3, etc 10s • compare two two digit numbers based on the meaning of the 10s and 1s and use symbols (<, >, =) MR.NOBT.1-2.3 Use place value understanding and properties of operations to add and subtract • add within 100 - add a two digit number and a one digit number, adding a two digit number and a multiple of 10, using models, drawings, strategies based on place value, properties of operations and the relationship between addition and subtraction. Connect to written. Understand that when adding making a ten is necessary • mentally find 10 more or 10 less than a given two digit number • subtract multiples of 10 from multiples of 10 using strategies listed above and connecting to written/explanation	• express the length of an object as a whole number of length units. Understand that the length of an object is measured by the same sized units, lined up with no gaps or overlaps.
	Counting and Cardinality	Operations and Algebraic Thinking	Number and Operations in Base Ten
K	MR.CC.K.1 Know number names and the count sequence • count to 100 by 1s and 10s • count forward from a given number (not 1)	MR.OAT.K.1 Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. • represent addition and subtraction with objects, fingers,	MR.NOBT.K.1 Work with numbers 11-19 to gain foundations for place value • compose and decompose numbers from 11-19 into tens and ones by using objects or drawings and record with drawing

	• write and represent numbers from 0-20 MR.CC.K.2 Count to tell the number of objects • understand the relationship between numbers and quantities • when counting, say number names in order, pairing each object with only one number • last number name said tells number of objects; number of objects is the same regardless of arrangement • each successive number is one larger • count to answer “how many” and count out a given number of objects MR.CC.K.3 Compare numbers • identify which group is less than, greater than or equal to by using matching and counting • compare two numbers between 1 and 10 presented as written numerals	mental images, drawings, sounds (claps), acting out situations, verbal explanations, expressions or equations • solve addition and subtraction word problems within 10, using drawings or objects • decompose numbers less than or equal to 10 into pairs in more than one way and write down pairs by drawing or equation • Find pairs that make 10 by using objects or drawing and record with drawings or equations • Fluently add and subtract within 5	or equation; understand that these numbers are composed of tens and ones
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